

Forecasting good volatility and bad volatility

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Riassunto: I rendimenti di molte attività finanziarie hanno distribuzioni asimmetriche, ma in letteratura il tema è solo marginalmente affrontato. La nostra congettura è che l'asimmetria sia dovuta ad una diversa dinamica dei rendimenti positivi rispetto a quelli negativi. In questa nota presentiamo un modello che consente di cogliere la diversa struttura dinamica, oltre alla diversa distribuzione, dei rendimenti positivi e negativi.

Keywords: Volatility, Skewness, GARCH, Asymmetric Dynamics

1. Introduction and motivation

It is quite common for financial assets returns to show conditional heteroskedasticity, leptokurtosis and skewness. The first two properties are usually dealt with GARCH or Stochastic Volatility models possibly with fat tailed distributions. However, skewness has received much less attention in literature. In fact, skewness is usually obtained as a byproduct of models for leverage effects such as the Exponential GARCH and the Threshold GARCH. In many cases, though, the standardized residuals of these models show significant skewness. Table 1 reports sample skewness and the relative significance for the daily percentage returns of ENEL and for the standardized residuals with respect to asymmetric GARCH with GED (conditional) distribution: skewness is even enhanced in the standardized residuals. Similarly, the absolute values of the minima are circa twice as large as the maxima, evidencing a significantly fatter negative tail.

	Mean	Median	Max	Min	St.Dev.	Skew	Kurt-3
Returns	0.004	0.000	6.266	-11.463	1.435	-0.604**	6.482**
EGARCH	0.002	0.000	4.435	-8.213	1.014	-0.735**	5.893**
TGARCH	0.001	0.000	3.200	-8.618	1.012	-0.869**	6.887**

Table 1: Daily ENEL returns from 1st Nov 1999 to 26th Feb 2007. Statistics of percentage returns and standardized residuals (** denotes significance at 1%).

Some authors have modelled skewness by using GARCH-type processes with conditional skewed distributions Harris *et al.* (2004); Lambert and Laurent (2001); Lanne and Saikkonen (2004). In this paper we pursue a different approach. We argue that the dynamics of negative returns may differ from that of positive returns and propose a class of processes that may be used to model these different behaviours.

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2. The model

Let r_t be the time series of returns. Define the series of positive and negative returns as

$$r_t^+ = r_t \cdot \mathbb{I}(r_t > 0), \quad r_t^- = r_t \cdot \mathbb{I}(r_t < 0)$$

with $\mathbb{I}(\cdot)$ indicator function. The returns are modelled as

$$r_t | r_t > 0 \sim D^+(h_t^+), \quad r_t | r_t < 0 \sim D^-(h_t^-)$$

where D^+ is a family of distributions with positive support and second non-central moment h_t^+ and D^- denotes a family of distributions with negative support and second non-central moment h_t^- . The two distributions may, of course, depend on other parameters, such as location and tail parameters. The conditional second moments are let evolve according to the following bivariate difference equation

$$\begin{bmatrix} h_t^+ \\ h_t^- \end{bmatrix} = \boldsymbol{\omega} + \mathbf{A} \begin{bmatrix} r_t^{+2} \\ r_t^{-2} \end{bmatrix} + \mathbf{B} \begin{bmatrix} h_{t-1}^+ \\ h_{t-1}^- \end{bmatrix}$$

with $\boldsymbol{\omega}$ (2×1) vector, $\mathbf{A}$ and $\mathbf{B}$ (2×2) matrices suitably restricted.

The model can be estimated by maximum likelihood and, nesting the simple GARCH(1,1), GARCH restrictions may be tested by means of a standard LR test.

The model has been applied to the daily returns of ENEL and the estimated good and bad volatilities are depicted in Figure 1.

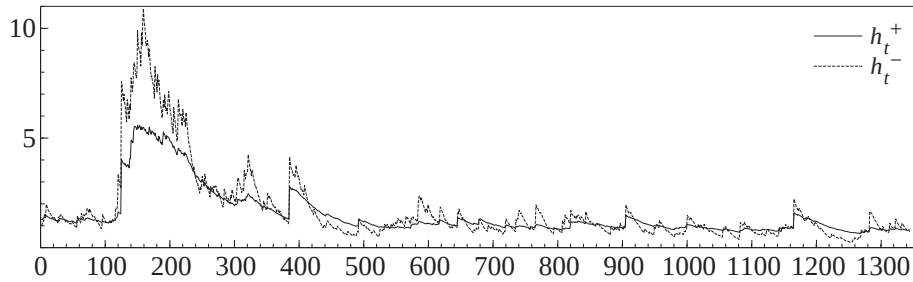


Figure 1: Positive and negative volatilities estimated on ENEL returns in the period 1st Jan 2001 – 26th Feb 2007.

References

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